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The photon–neutrino interaction induced by non-commutativity and astrophysical bounds

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Abstract. In this work we propose a possible mechanism of left- and right-handed neutrino couplings to photons, which arises quite naturally in non-commutative field theory. We estimate the predicted additional energy-loss in stars induced by space-time non-commutativity. The usual requirement that any new energy-loss mechanism in globular stellar clusters should not excessively exceed the standard neutrino losses implies a scale of non-commutative gauge theory above the scale of weak interactions.

1 Introduction

Neutrinos do not carry a U(1) (electromagnetic) charge and hence do not directly couple to Abelian gauge bosons (photons) – at least not in a commutative setting. In the presence of space-time non-commutativity, it is, however, possible to couple neutral particles to gauge bosons via a star commutator. The relevant covariant derivative is

$$\widehat{D}_\mu \widehat{\psi} = \partial_\mu \widehat{\psi} - \mathrm{i}\kappa e \widehat{A}_\mu \star \widehat{\psi} + \mathrm{i}\kappa e \widehat{\psi} \star \widehat{A}_\mu \,, \qquad (1)$$

with the $\star$-product and a coupling constant κe that corresponds to a multiple (or fraction) κ of the positron charge e. The $\star$-product is associative but, in general, not commutative – otherwise the proposed coupling to the non-commutative photon field $\widehat{A}_\mu$ would of course be zero. We choose a perturbative approach to NC gauge field theory in which the action, like the $\star$-product itself, is expanded in powers of a Poisson tensor $\theta^{\mu\nu}$. A discussion of this approach and a critical comparison with the original $\star$-product approach is given in Sect. 3. In (1), one may think of the non-commutative neutrino field $\widehat{\psi}$ as having left charge $+\kappa e$, right charge $-\kappa e$ and total charge zero. From the perspective of non-Abelian gauge theory, one could also say that the neutrino field is charged in a non-commutative analogue of the adjoint representation with the matrix multiplication replaced by the $\star$-product. From a geometric point of view, photons do not directly couple to the "bare" commutative neutrino fields, but rather modify the non-commutative background. The neutrinos propagate in that background.

Kinematically, a decay of photons into neutrinos is, of course, allowed only for off-shell photons. This is still true in a constant or sufficiently slowly varying non-commutative background: Such a background does not lead to a violation of four-momentum conservation, although it may break other Lorentz symmetries. Physically, such a coupling of neutral particles to gauge bosons is possible because the non-commutative background is described by an antisymmetric tensor $\theta^{\mu\nu}$ that essentially plays the role of an external field in the theory [1–15]. The $\star$-product in (1) is a (non-local) bilinear expression in the fields and their derivatives that takes the form of a series in $\theta^{\mu\nu}$. A similar expansion (Seiberg–Witten map) exists for the non-commutative fields $\widehat{\psi}$, $\widehat{A}_\mu$ in terms of $\theta^{\mu\nu}$, ordinary "commutative" fields ψ, A_μ and their derivatives. For related work on non-commutative field theory and phenomenology, see [14–18]. To lowest order in θ the covariant derivative is

$$\widehat{D}_\mu \widehat{\psi} = \partial_\mu \widehat{\psi} + \kappa e \theta^{\nu\rho}\, \partial_\nu \widehat{A}_\mu\, \partial_\rho \widehat{\psi} \,.$$

Following [16], the scale of non-commutativity Λ_{NC} is fixed by choosing dimensionless matrix elements $c^{\mu\nu} = \Lambda_{\mathrm{NC}}^2 \theta^{\mu\nu}$ of order one. We shall assume $c_{\mu\nu} c^{\mu\nu} > 0$ to avoid a discussion of potential difficulties with unitarity in non-commutative QFT.

Gauge invariance requires that all e's in the action should be multiplied by κ. To the order considered in this letter, κ can be absorbed in a rescaling of θ, i.e. a rescaling of the definition of Λ_{NC}. It should, however, be noted that on purely phenomenological grounds it would be natural if $(e\kappa)^2/4\pi$ was a running coupling constant just like α_{em} that increases with energy. Technically, we are unfortunately not (yet) in a position to compute such a running.

2 The model

The coupling (1) is part of an effective model of particle physics involving neutrinos and photons on non-commutative space-time. It describes the scattering of particles that enter from an asymptotically commutative region into a non-commutative interaction region. The model satisfies the following requirements [1–15].

(i) Non-commutative effects are described perturbatively. The action is written in terms of asymptotic commutative fields.

(ii) The action is gauge invariant under U(1)-gauge transformations.

(iii) It is possible to extend the model to a non-commutative electroweak model based on the gauge group U(1)×SU(2). An appropriate non-commutative electroweak model with coupling $\kappa = 1$ can in fact be constructed with the same tools that were used for the non-commutative standard model of [11].[1]

The action of such an effective model differs from the commutative theory essentially by the presence of star products and the expansion of fields via Seiberg–Witten (SW) maps. The Seiberg–Witten maps [8] are necessary to express the non-commutative fields $\widehat{\psi}$, $\widehat{A}_\mu$ that appear in the action and transform under non-commutative gauge transformations, in terms of their asymptotic commutative counterparts ψ and A_μ. The coupling of matter fields to Abelian gauge bosons is a non-commutative analogue of the usual minimal coupling scheme.

The action for a neutral fermion that couples to an Abelian gauge boson in a non-commutative background is

$$S = \int \mathrm{d}^4 x \left(\overline{\widehat{\psi}} \star \mathrm{i}\gamma^\mu \widehat{D}_\mu \widehat{\psi} - m \overline{\widehat{\psi}} \star \widehat{\psi} \right). \tag{2}$$

Here $\widehat{\psi}_{\left(\substack{\mathrm{L}\\ \mathrm{R}}\right)} = \psi_{\left(\substack{\mathrm{L}\\ \mathrm{R}}\right)} + e\theta^{\nu\rho} A_\rho \partial_\nu \psi_{\left(\substack{\mathrm{L}\\ \mathrm{R}}\right)}$ and $\widehat{A}_\mu = A_\mu + e\theta^{\rho\nu} A_\nu \times \left[\partial_\rho A_\mu - \frac{1}{2}\partial_\mu A_\rho\right]$ is the Abelian NC gauge potential expanded by the Seiberg–Witten map.[2]

To first order in θ, the gauge-invariant action reads

$$\begin{aligned} S = \int \mathrm{d}^4 x\, &\Big\{ \bar{\psi} \left[\mathrm{i}\gamma^\mu \partial_\mu - m \left(1 - \frac{e}{2} \theta^{\mu\nu} F_{\mu\nu} \right) \right] \psi \\ &+ \mathrm{i}e\theta^{\mu\nu} \left[\left(\partial_\mu \bar{\psi} \right) A_\nu \gamma^\rho \left(\partial_\rho \psi \right) - \left(\partial_\rho \bar{\psi} \right) A_\nu \gamma^\rho \left(\partial_\mu \psi \right) \right. \\ &\left. + \bar{\psi} \left(\partial_\mu A_\rho \right) \gamma^\rho \left(\partial_\nu \psi \right) \right] \Big\} . \end{aligned} \tag{3}$$

Integrating by parts, (3) becomes manifestly gauge invariant and can be conveniently expressed by

$$\begin{aligned} S = \int \mathrm{d}^4 x\, \bar{\psi} \Big[&\left(\mathrm{i}\gamma^\mu \partial_\mu - m \right) \\ &- \frac{e}{2} F_{\mu\nu} \left(\mathrm{i}\, \theta^{\mu\nu\rho} \partial_\rho - \theta^{\mu\nu} m \right) \Big] \psi \\ \equiv \int \mathrm{d}^4 x\, \bar{\psi} \Big[&\left(\mathrm{i}\gamma^\mu \partial_\mu - m \right) \\ &- \frac{e}{2} \theta^{\nu\rho} \left(\mathrm{i}\gamma^\mu \left(F_{\nu\rho} \partial_\mu + F_{\mu\nu} \partial_\rho + F_{\rho\mu} \partial_\nu \right) - m F_{\nu\rho} \right) \Big] \psi, \end{aligned} \tag{4}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\theta^{\mu\nu\rho} = \theta^{\mu\nu}\gamma^\rho + \theta^{\nu\rho}\gamma^\mu + \theta^{\rho\mu}\gamma^\nu$.

The above action presents a tree-level interaction of photons and neutrinos on non-commutative space-time.

It is interesting to note that we can write

$$\begin{aligned} \mathrm{i}\bar{\psi} F_{\mu\nu}\, \theta^{\mu\nu\rho} \partial_\rho \psi &= F_{\mu\nu} \left(\theta^{\mu\nu} T^\rho{}_\rho + \theta^{\nu\rho} T^\mu{}_\rho + \theta^{\rho\mu} T^\nu{}_\rho \right) \\ &\equiv \theta^{\mu\nu} \left(T^\rho{}_\mu F_{\nu\rho} + T^\rho{}_\nu F_{\rho\mu} + T\,{}_\rho^{\;rho} F_{\mu\nu} \right), \end{aligned} \tag{5}$$

where

$$T^{\mu\nu} = \mathrm{i}\bar{\psi}\gamma^\mu \partial^\nu \psi \tag{6}$$

represents the stress–energy tensor of commutative gauge theory for free fermion fields [19]. Hence, for the massless case (4) reduces to the coupling between the stress–energy tensor of the neutrino $T^{\mu\nu}$ and the symmetric tensor composed by θ and F. This nicely illustrates our assertion that we are seeing the interaction of the neutrino with a modified photon–θ background.

So far we have not discussed how the terms of the action (2) that we have introduced can be embedded into a model of the full non-commutative electroweak sector. We have instead focused on the interaction term that is relevant for the computation of the plasmon decay rate. In particular we have not yet discussed the form of the gauge kinetic term. Since the choice of model has some bearing on the resulting phenomenology, in particular in the infrared, we shall give a brief overview and discussion of various approaches to non-commutative gauge theory. All have in common that the action resembles Yang–Mills theory, with matrix multiplication replaced by $\star$-products.

3 Approaches to NC gauge theory

The approach to non-commutative gauge theory that we use belongs to a class of models that expand the action in θ before quantization [9–15]. Here we do not necessarily have infrared problems, UV/IR mixing or a negative beta function. The approach very nicely captures new interactions and violations of space-time symmetries induced by non-commutativity. Furthermore it can be directly applied to realistic gauge groups like U(1)×SU(2) in the present case.

The infrared deserves some discussion, because we could in principle have neutrino condensation in our model. For pure non-commutative Maxwell theory the photon self-energy has been computed to all loop orders in [28], the beta function is that of ordinary Abelian gauge theory. For neutrinos in the $\star$-adjoint representation we do not expect any contribution to the beta function up to the second order in θ, considering the relevant terms that may enter in the computation of the beta function at that order. The

[1] For a model in which only the neutrino has dual left and right charges, $\kappa = 1$ is required by the gauge invariance of the action.

[2] Note that instead of the Seiberg–Witten map of Dirac fermions ψ one can consider a “chiral” SW map. This SW map is compatible with grand unified models where fermion multiplets are chiral [12].

computation of higher order corrections to the beta function in our model is an open project, but the expected result is a theory without infrared problems and in particular without neutrino condensation. In fact, a negative beta function can be completely avoided in NC QED in the SW map approach: We can choose a reducible representation of the U(1) gauge group that avoids non-commutativity-induced self-interaction terms of an odd number of photons and nevertheless leads to ordinary QED in the limit $\theta \to 0$. An objection to the θ-expanded approach is that it is not clear how it can capture non-perturbative information about the non-commutativity of space-time, since this would require summation to all orders in θ.

In contrast to our model, the original $\star$-product approach to non-commutative U(1), is defined in terms of Feynman rules that are directly obtained from the action (in momentum space) without first expanding the $\star$-products or fields in terms of θ. The resulting phase factors play the role of structure constants in ordinary "commutative" Yang–Mills theory. The result is that the beta function resembles that of a non-Abelian gauge theory even though the structure group is Abelian [20]. The beta function with matter in the adjoint has been computed in this approach in [21]; see also [22]. The beta function is negative and we would expect problems in the infrared if we were to take this theory at face value even at low energies. In particular there could be condensation of neutrino–antineutrino pairs and one could question whether it is justified to work in the tree-level approximation. There is also UV/IR mixing.[3] A $N = 4$ supersymmetric extension of the model, softly broken down to $N = 0$, has been considered with the goal to get a more realistic phenomenology, but was also not satisfactory [24]. Infrared problems could perhaps be avoided with more sophisticate quantization and renormalization procedures [25]. Since the approach is limited to $U(N)$ gauge groups in the fundamental representation, the trick that can be used to avoid triple gauge boson couplings in the SW map approach cannot be used here. The original approach to non-commutative gauge theory has other problems already at the classical level: Charges of the particles are limited to $\pm e$ or zero, in clear conflict with the known particle spectrum. (See however [26], where the charges are still quantized, but to the correct values of the usual quarks and leptons.) The fields do not transform covariantly under general coordinate transformations [27] and there are problems with renormalizability [25]. The original approach to NC QED is thus inconsistent with experimental facts, at both the classical and the quantum level [20–28].

The quantization of time–space NC field theory in the original $\star$-product approach has been extensively discussed in [30]. A similar investigation of NCGT in the SW-map approach still needs to be done. It is expected to lead to more encouraging conclusions.

[3] This is not necessarily a bad thing: UV/IR mixing effects in non-commutative gauge theory on D-branes can capture information about the closed string spectrum of the parent string theory [23].

Concerning the physics to be investigated, the picture that we have in mind is that of a space-time that has a continuous "commutative" description at low energies and long distances, but a non-commutative structure at high energies and short distances. There could be some kind of phase-transition involved. At high energies we can model space-time using $\star$-products. This description is expected not to be valid at low energies. The technical consequence is that we expand up to a certain order in θ and considering renormalization of this truncated theory up to the same order in θ. It is obvious that in this truncated theory there will not arise any infrared problem. This reflects very well our assumption: At low energies and large distances the non-commutative theory has to be modified.

Our model is meant to provide an effective description of space-time non-commutativity involving the photon–neutrino contact interaction. Therefore, we treat our action as an effective action, disregarding renormalizability in the ordinary sense. This approach is similar to chiral dynamics in pion physics. As we have discussed above, it differs fundamentally from other approaches based on $\star$-products that are not θ-expanded and do not use the Seiberg–Witten map: We expand the action up to a certain fixed order in θ *before* quantization. The effective theory obtained appears to be anomaly free [29].

4 Plasmon decay and astrophysical bound

We now apply our model to the decay of plasmons into neutrino–antineutrino pairs that would be induced by a hypothetical stellar non-commutative space-time structure. The resulting neutrinos can escape from the star and thereby lead to an energy-loss. To obtain the "transverse plasmon" decay rate in stars on the scale of non-commutativity, we start with the action determining the $\gamma\nu\bar{\nu}$ interaction. From (3) we extract, for left or right and possibly massive neutrinos, the following Feynman rule for the gauge-invariant $\gamma(q) \to \nu(k')\bar{\nu}(k)$ vertex in momentum space:

$$\begin{aligned}&\Gamma^{\mu}_{\binom{\mathrm{L}}{\mathrm{R}}}(\nu\bar{\nu}\gamma) \qquad (7)\\ &= \mathrm{i}e\frac{1}{2}(1 \mp \gamma_5)\left[(q\theta k)\gamma^{\mu} + (\not{k} - m_{\nu})\widetilde{q}^{\mu} - \not{q}\widetilde{k}^{\mu}\right].\end{aligned}$$

Here we have used the notation $\widetilde{q}^{\mu} \equiv \theta^{\mu\nu}q_{\nu}$, $\widetilde{k}^{\mu} \equiv \theta^{\mu\nu}k_{\nu}$. In the case of massless neutrinos, the vertex (7) becomes symmetric:

$$\Gamma^{\mu}_{\binom{\mathrm{L}}{\mathrm{R}}}(\nu\bar{\nu}\gamma) = \mathrm{i}e\frac{1}{2}(1 \mp \gamma_5)\theta^{\mu\nu\rho}k_{\nu}q_{\rho}. \qquad (8)$$

In stellar plasma, the dispersion relation of photons is identical with that of a massive particle [31–33]:

$$q^2 \equiv \mathrm{E}_{\gamma}^2 - \mathbf{q}_{\gamma}^2 = \omega_{\mathrm{pl}}^2 \qquad (9)$$

with ω_{pl} being the plasma frequency.

From the gauge-invariant amplitude $\mathcal{M}_{\gamma\nu\bar{\nu}}$ in momentum space for the plasmon (off-shell photon) decay to the left-handed ν (right-handed $\bar{\nu}$) and/or right-handed ν (left-handed $\bar{\nu}$), if "existing" with the same mass $\to 0$, massive neutrinos in our model, we have[4]

$$\sum_{\text{pol.}} |\mathcal{M}_{\gamma\nu\bar{\nu}}|^2$$

$$= 4e^2 \left[\left(q^2 - 2m_\nu^2\right)\left(m_\nu^2 \widetilde{q}^2 - (q\theta k)^2\right) + m_\nu^2 q^2 \left(\widetilde{k}^2 - \widetilde{k}\widetilde{q}\right)\right].$$

Phase-space integration of this expression then gives

$$\Gamma\left(\gamma_{\mathrm{pl}} \to \bar{\nu}_{\binom{\mathrm{L}}{\mathrm{R}}}\nu_{\binom{\mathrm{L}}{\mathrm{R}}}\right) = \frac{\alpha}{48}\frac{\omega_{\mathrm{pl}}^6}{\mathrm{E}_\gamma \Lambda_{\mathrm{NC}}^4}\sqrt{1 - 4\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2}}$$

$$\times \left[\left(1 + 2\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2} - 12\frac{m_\nu^4}{\omega_{\mathrm{pl}}^4}\right)\sum_{i=1}^{3}\left(c^{0i}\right)^2 \right.$$

$$\left. + 2\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2}\left(1 - 4\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2}\right)\sum_{\substack{i,j=1\\ i<j}}^{3}\left(c^{ij}\right)^2 \right]. \qquad (10)$$

In the above formula we have parametrized the c_{0i}'s by introducing the angles characterizing the background $\theta^{\mu\nu}$ field of the theory [16]:

$$c_{01} = \cos\xi, \quad c_{02} = \sin\xi\ \cos\zeta, \quad c_{03} = \sin\xi\ \sin\zeta,$$

where ξ is the angle between the $\mathbf{E}_\theta$ field and the direction of the incident beam, i.e. the photon axes. The angle ζ defines the origin of the ϕ axis. The c_{0i}'s are not independent; in pulling out the overall scale Λ_{NC}, we can always impose the constraint $\mathbf{E}_\theta^2 \equiv \sum_{i=1}^{3}(c^{0i})^2 = 1$. Here we consider three physical cases: $\xi = 0,\ \pi/4,\ \pi/2$, which for $\zeta = \pi/2$ satisfy the imposed constraint. This parametrization provides a good physical interpretation of the NC effects [16].

In the rest frame of the medium, the decay rate of a "transverse plasmon", of energy E_γ for the left–left and/or right–right massless neutrinos and for the constraint $\mathbf{E}_\theta^2 = 1$, is given by

$$\Gamma_{\mathrm{NC}}\left(\gamma_{\mathrm{pl}} \to \nu_{\binom{\mathrm{L}}{\mathrm{R}}}\bar{\nu}_{\binom{\mathrm{L}}{\mathrm{R}}}\right) = \frac{\alpha}{48}\frac{1}{\Lambda_{\mathrm{NC}}^4}\frac{\omega_{\mathrm{pl}}^6}{\mathrm{E}_\gamma}. \qquad (11)$$

The standard model (SM) photon–neutrino interaction at tree level does not exist. However, the effective photon–neutrino–neutrino vertex $\Gamma_{\mathrm{eff}}^{\mu}(\gamma\nu\bar{\nu})$ is generated through 1-loop diagrams, which are very well known in heavy-quark physics as "penguin diagrams". Such effective interactions give non-zero charge radius, as well as the contribution to the "transverse plasmon" decay rate [34–37]. For details, see [36]. Finally, note that the dipole moment operator $\sim e m_\nu G_{\mathrm{F}} \bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}$, also generated by the "neutrino-penguin diagram", gives very small contributions to the decay rate because of the smallness of the neutrino mass, i.e. $m_\nu < 1\,\mathrm{eV}$ [38].

[4] Note that this result is independent on different choices of the Seiberg–Witten map for right-handed Dirac fermions; see footnote 2.

The corresponding SM neutrino-penguin-loop result for the "transverse plasmon" decay rate is [36]

$$\Gamma_{\mathrm{SM}}\left(\gamma_{\mathrm{pl}} \to \nu_{\mathrm{L}}\bar{\nu}_{\mathrm{L}}\right) = \frac{c_{\mathrm{v}}^2 G_{\mathrm{F}}^2}{48\pi^2\alpha}\ \frac{\omega_{\mathrm{pl}}^6}{\mathrm{E}_\gamma}. \qquad (12)$$

For ν_e, we have $c_{\mathrm{v}} = \frac{1}{2} + 2\sin^2\Theta_{\mathrm{W}}$, while for ν_μ and ν_τ we have $c_{\mathrm{v}} = -\frac{1}{2} + 2\sin^2\Theta_{\mathrm{W}}$. Comparing the rate of the decays into all three neutrino families, we thus need to include a factor of 3 for the NC result, while $c_{\mathrm{v}}^2 = 0.79$ for the SM result [39]. From the ratio of the rates

$$\Re \equiv \frac{\sum_{\mathrm{flavours}} \Gamma_{\mathrm{NC}}\left(\gamma_{\mathrm{pl}} \to \nu_{\mathrm{L}}\bar{\nu}_{\mathrm{L}} + \nu_{\mathrm{R}}\bar{\nu}_{\mathrm{R}}\right)}{\sum_{\mathrm{flavours}} \Gamma_{\mathrm{SM}}\left(\gamma_{\mathrm{pl}} \to \nu_{\mathrm{L}}\bar{\nu}_{\mathrm{L}}\right)} = \frac{6\pi^2\alpha^2}{c_{\mathrm{v}}^2 G_{\mathrm{F}}^2 \Lambda_{\mathrm{NC}}^4}, \qquad (13)$$

we obtain

$$\Lambda_{\mathrm{NC}} = \frac{80.8}{\Re^{1/4}}\ (\mathrm{GeV}). \qquad (14)$$

A standard argument involving globular cluster stars tells us that any new energy-loss mechanism must not excessively exceed the standard neutrino losses; see Sect. 3.1 in [40]. Expressed in another way, we should approximately require $\Re < 1$, translating into

$$\Lambda_{\mathrm{NC}} > \left(\frac{6\pi^2\alpha^2}{c_{\mathrm{v}}^2 G_{\mathrm{F}}^2}\right)^{1/4} \cong 81\,\mathrm{GeV}. \qquad (15)$$

If sterile neutrinos (ν_{R}) do not exist, the scale of non-commutativity is approximately $\Lambda_{\mathrm{NC}} > 68$ GeV.

5 Conclusion

We have proposed a way in which neutrinos can couple to photons via mutual interactions with a hypothetical non-commutative structure of space-time. For the construction of the model we choose a perturbative approach to non-commutative gauge theory based on the Seiberg–Witten map. We explain the reasons for that choice and discuss the drawbacks of other possibilities that have appeared in the literature. The model is applied to the decay of plasmons into neutrinos. Bounds on the scale of non-commutativity are obtained by limits on the implied energy-loss in stars.

In our model of non-commutativity-induced anomalous $\gamma\nu\bar{\nu}$ interaction, photons are also coupled to sterile neutrinos in the same, U(1)-gauge-invariant, way as the left-handed ones, contrary to the situation in the standard model. The electromagnetic gauge invariance of the $\gamma\nu\bar{\nu}$ amplitude comes automatically, since the starting action is manifestly U(1) gauge invariant. The interaction (3) produces extra contributions relative to the SM in the non-commutative background.

The non-commutativity scale depends on the requirement $\Re < 1$ and from this aspect, the constraint $\Lambda_{\mathrm{NC}} > 80\,\mathrm{GeV}$, obtained from the energy-loss in globular stellar clusters through the SM calculation, represents the lower bound on the scale of non-commutative gauge field theories. If we instead use the $\Lambda_{\mathrm{NC}} \gtrsim 1$ TeV [16], this would result in $\Re \lesssim 10^{-4}$ (in accord with astrophysical bounds on plasmon decay and neutrino magnetic moments [41, 42]), producing a very small contribution to the cooling processes in globular stellar clusters.

The actual non-commutativity scale also depends on the strength of the non-commutative coupling constant which we have absorbed into θ. Phenomenologically, it would be natural if $\alpha_\kappa \equiv (\kappa e)^2/4\pi$ were a running coupling constant. This could account at least for some of the discrepancy between the limits that we have obtained here and the ones that we have found in [43], where we study the dipole moments that are induced by non-commutativity-induced photon-neutrino interaction.

The bound that we have obtained is relatively low, however, it is based on a completely new interaction channel and a completely different "laboratory" than other constraints and as such appears worth communicating. The bound is of comparable order of magnitude as the first limits on NCQED obtained from collider experiments that were recently presented by the OPAL Collaboration [44]. There no significant deviation from the SM prediction was found and at the 95% confidence level the limit on the non-commutative scale was set at $\Lambda_{\mathrm{NC}} > 141\,\mathrm{GeV}$.

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